WAVE PARTICLE DUALITY

Evidence for wave-particle duality

- Photoelectric effect
- Compton effect

- Electron diffraction
- Interference of matter-waves

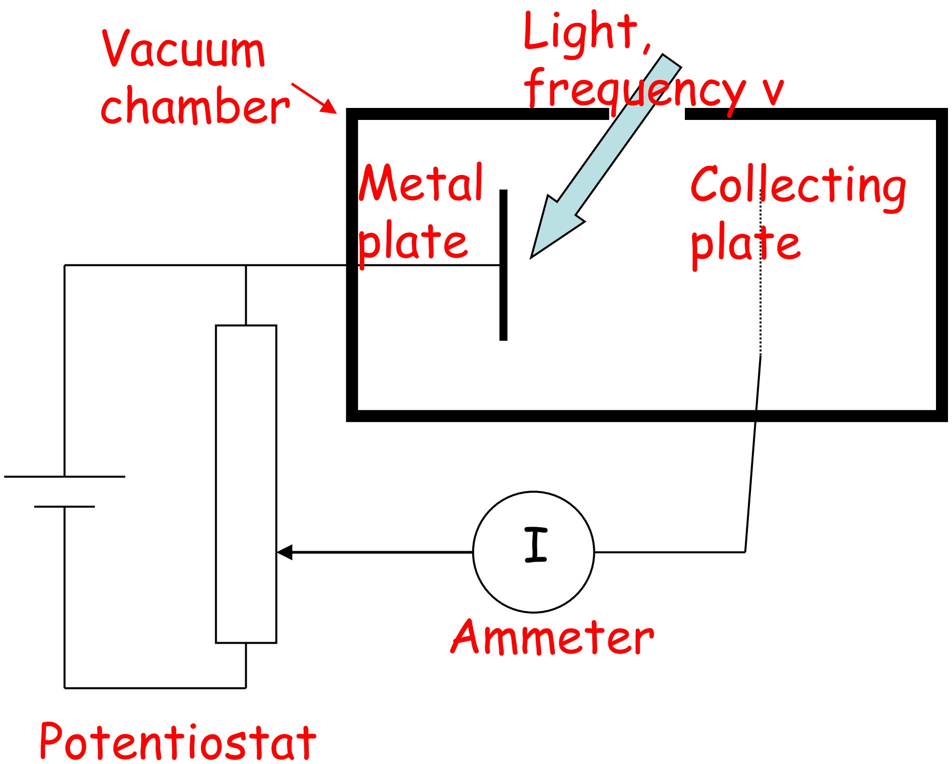
Consequence: Heisenberg uncertainty principle

PHOTOELECTRIC EFFECT

Hertz

J.J. Thomson

When UV light is shone on a metal plate in a vacuum, it emits charged particles (Hertz 1887), which were later shown to be electrons by J.J. Thomson (1899).



Classical expectations

Electric field E of light exerts force $\mathbf{F}=-e\mathbf{E}$ on electrons. As intensity of light increases, force increases, so KE of ejected electrons should increase. Electrons should be emitted whatever the frequency ν of the light, so long as **E** is sufficiently large

For very low intensities, expect a time lag between light exposure and emission, while electrons absorb enough energy to escape from material

PHOTOELECTRIC EFFECT (cont)

Actual results:

Maximum KE of ejected electrons is independent of intensity, but dependent on ν

For $\nu<\nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted

There is no time lag.

However, rate of ejection of electrons depends on light intensity.

Einstein's interpretation (1905):

Light comes in packets of energy (*photons*) $E = h\nu$

An electron absorbs a single photon to leave the material

Einstein

Millikan

The maximum KE of an emitted electron is then

$$K_{\max} = h\nu - W$$

Planck constant: universal constant of nature

$h = 6.63\times10^{-34}\,\mathrm{Js}$

Work function: minimum energy needed for electron to escape from metal (depends on material, but usually 2-5eV)

Verified in detail through subsequent experiments by Millikan

SUMMARY OF PHOTON PROPERTIES

Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Also have relation between momentum and wavelength

Relativistic formula relating energy and momentum ⇒ $$E^2 = p^2c^2 + m^2c^4$$

For light $E = pc$ and $c = \lambda\nu$

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

Also commonly write these as

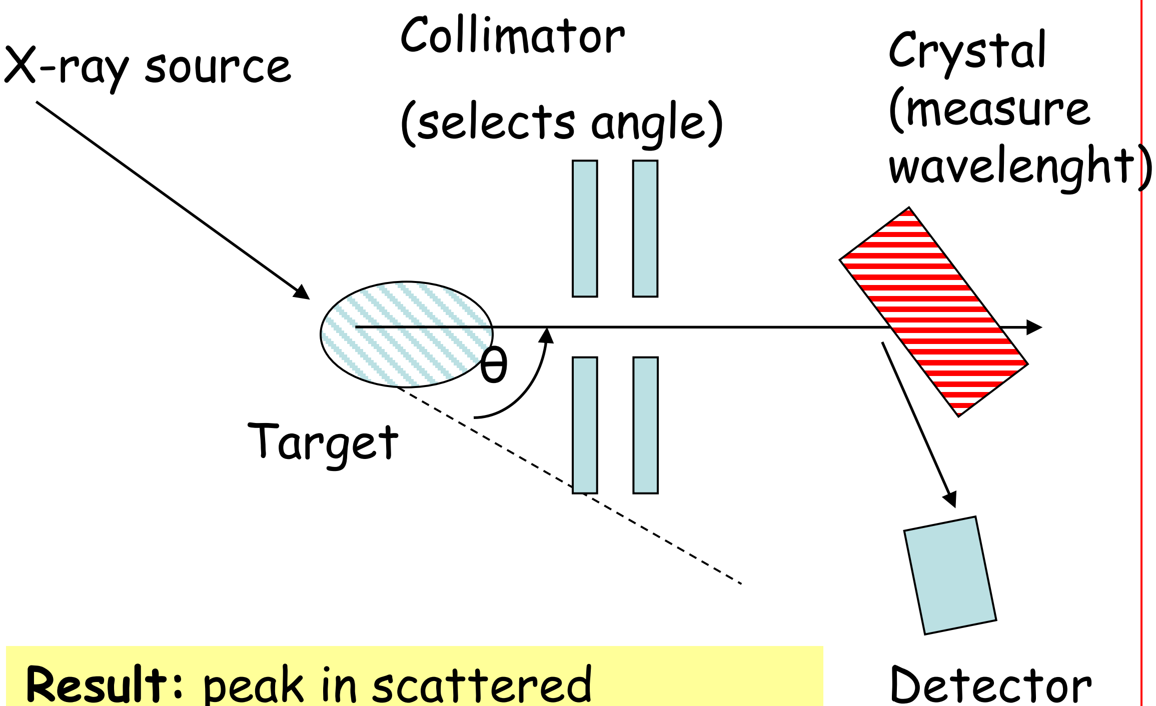
$$E = \hbar\omega \qquad p = \hbar k \qquad \omega = 2\pi\nu \qquad k = \frac{2\pi}{\lambda} \qquad \hbar = \frac{h}{2\pi}$$

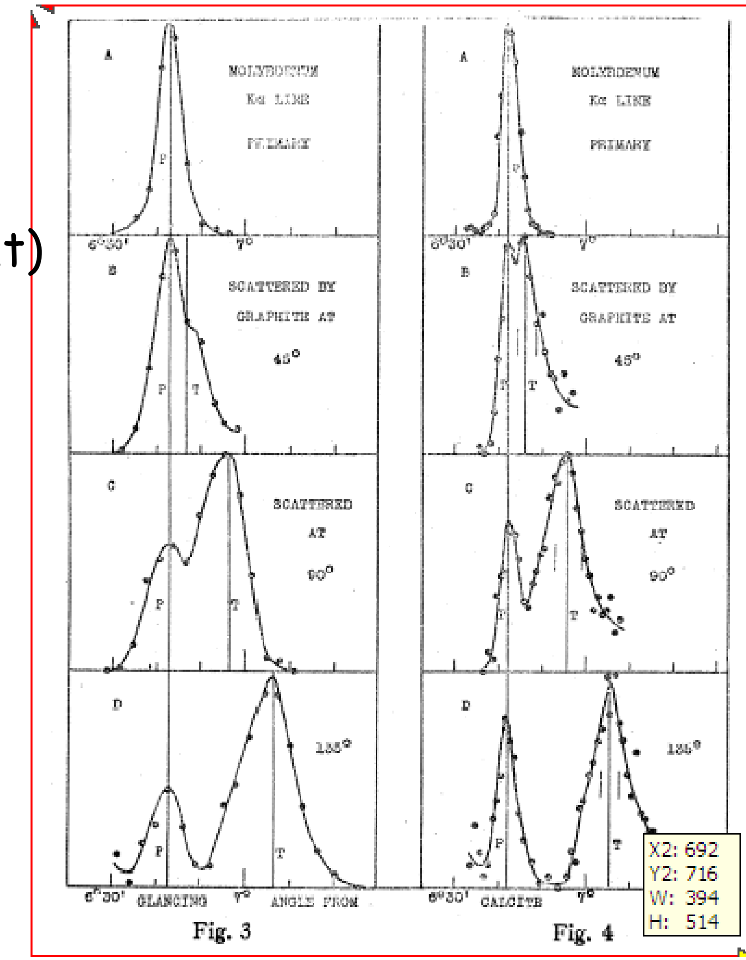
angular frequency (ω), wavevector (k), hbar ($\hbar$)

COMPTON SCATTERING

Compton

Compton (1923) measured intensity of scattered X-rays from solid target, as function of wavelength for different angles. He won the 1927 Nobel prize.





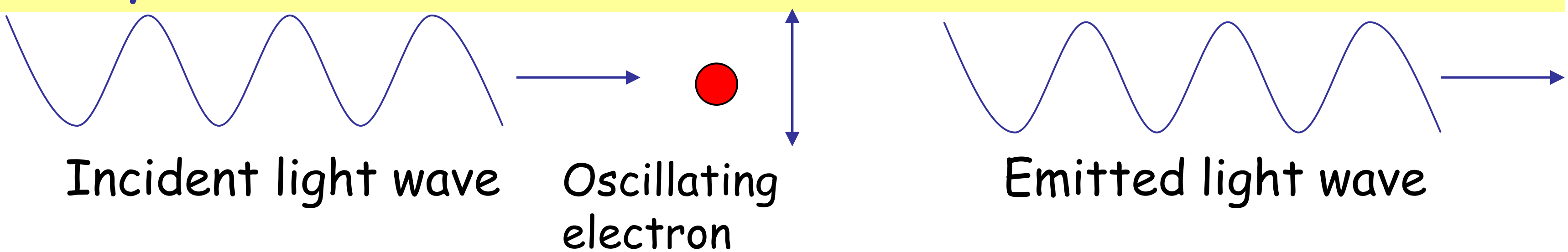
Result: peak in scattered radiation shifts to longer wavelength than source. Amount depends on θ (but not on the target material).

A.H. Compton, *Phys. Rev.* **22** 409 (1923)

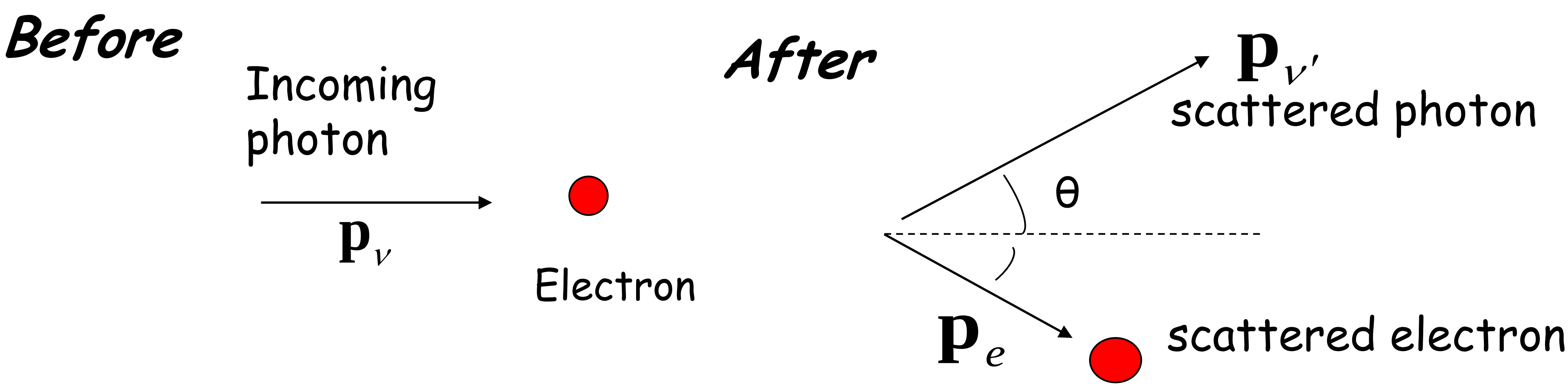
COMPTON SCATTERING (cont)

Classical picture: oscillating electromagnetic field causes oscillations in positions of charged particles, which re-radiate in all directions at *same frequency and wavelength* as incident radiation.

Change in wavelength of scattered light is completely unexpected classically

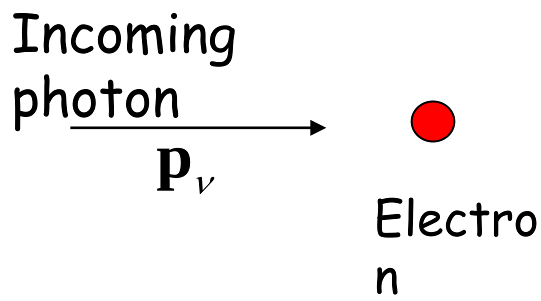


Compton's explanation: "billiard ball" collisions between particles of light (X-ray photons) and electrons in the material

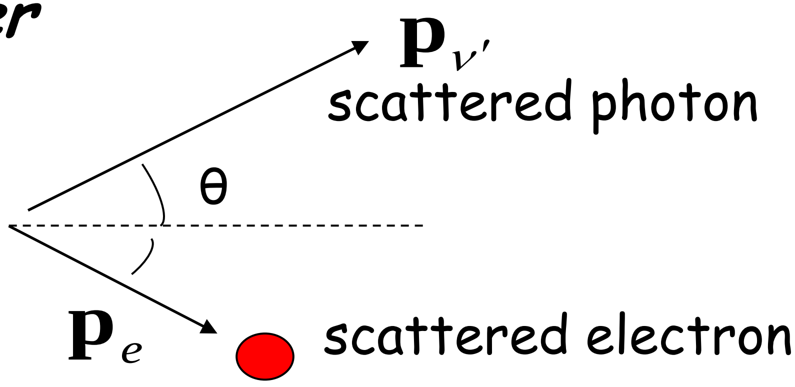


COMPTON SCATTERING (cont)

Before



After



Conservation of energy

$$h\nu + m_e c^2 = h\nu' + \left(p_e^2 c^2 + m_e^2 c^4\right)^{1/2}$$

Conservation of momentum

$$\mathbf{p}_\nu = \frac{h}{\lambda}\hat{\mathbf{i}} = \mathbf{p}_{\nu'} + \mathbf{p}_e$$

From this Compton derived the change in wavelength

$$\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$$
$$= \lambda_c\left(1 - \cos\theta\right) \geq 0$$

$$\lambda_c = \text{ Compton wavelength } = \frac{h}{m_e c} = 2.4 \times 10^{-12}\,\text{m}$$

COMPTON SCATTERING (cont)

Note that, at all angles there is also an unshifted peak.

This comes from a collision between the X-ray photon and the nucleus of the atom

$$\lambda' - \lambda = \frac{h}{m_N c}(1 - \cos\theta) > 0$$

since $m_N > m_e$

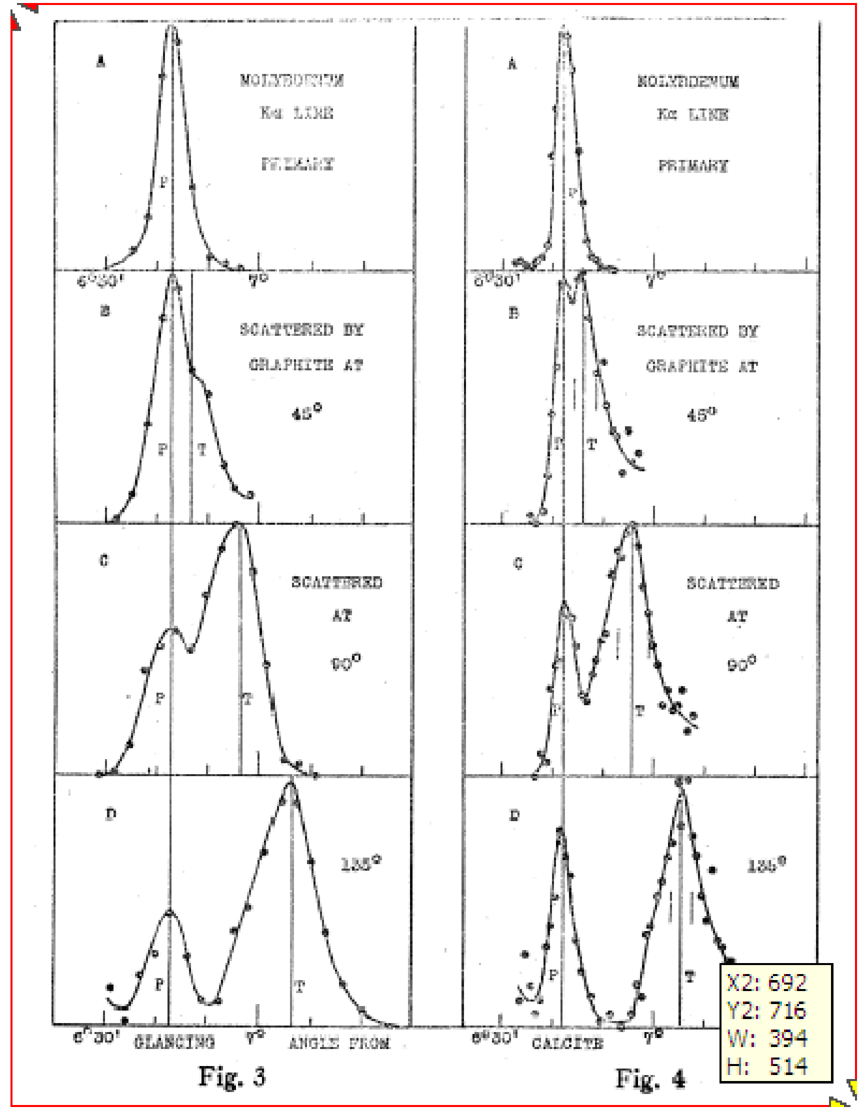


Fig. 3 Fig. 4

WAVE-PARTICLE DUALITY OF LIGHT

In 1924 Einstein wrote:- " There are therefore now two theories of light, both indispensable, and … without any logical connection."

Evidence for wave-nature of light

- Diffraction and interference

Evidence for particle-nature of light

- Photoelectric effect
- Compton effect

- Light exhibits diffraction and interference phenomena that are *only* explicable in terms of wave properties
- Light is always detected as packets (photons); if we look, we never observe half a photon
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength)

De Broglie

MATTER WAVES

We have seen that light comes in discrete units (photons) with particle properties (energy and momentum) that are related to the wave-like properties of frequency and wavelength.

In 1923 Prince Louis de Broglie postulated that ordinary matter can have wave-like properties, with the wavelength λ related to momentum p in the same way as for light

de Broglie relation

$$\lambda = \frac{h}{p}$$

de Broglie wavelength

Planck's constant

$$h = 6.63 \times 10^{-34}\ \text{Js}$$

wavelength depends on momentum, not on the physical size of the particle

Prediction: We should see diffraction and interference of matter waves

Estimate some de Broglie wavelengths

- Wavelength of electron with 50eV kinetic energy

$$K = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} \Rightarrow \lambda = \frac{h}{\sqrt{2m_e K}} = 1.7\times10^{-10}\,\mathrm{m}$$

- Wavelength of Nitrogen molecule at room temp.

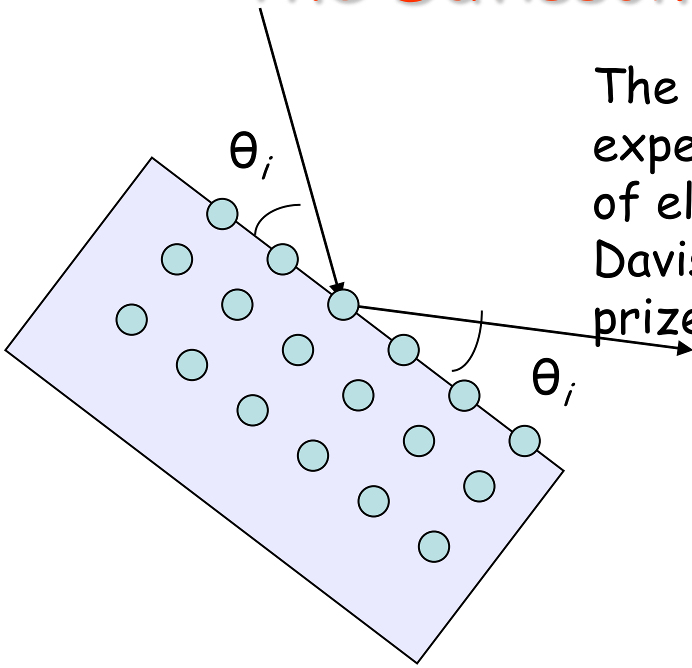
$$K = \frac{3kT}{2}, \quad \mathrm{Mass} = 28\mathrm{m_u}$$

$$\lambda = \frac{h}{\sqrt{3MkT}} = 2.8\times10^{-11}\,\mathrm{m}$$

- Wavelength of Rubidium(87) atom at 50nK

$$\lambda = \frac{h}{\sqrt{3MkT}} = 1.2\times10^{-6}\,\mathrm{m}$$

ELECTRON DIFFRACTION
The Davisson-Germer experiment (1927)

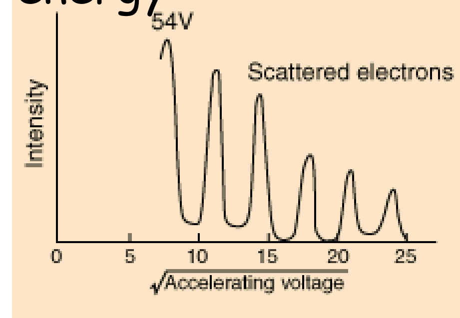


The Davisson-Germer experiment: scattering a beam of electrons from a Ni crystal. Davisson got the 1937 Nobel prize.

Davisson

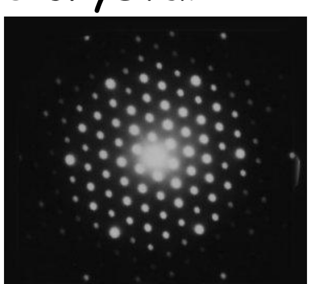
G.P. Thomson

At fixed *angle*, find sharp peaks in intensity as a function of electron energy



Davisson, C. J., "Are Electrons Waves?," Franklin Institute Journal **205**, 597 (1928)

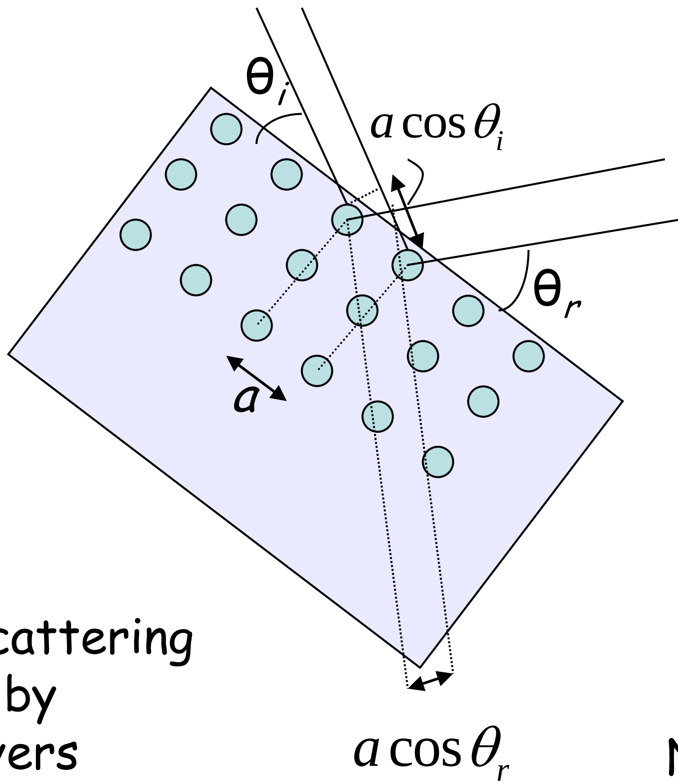
At fixed accelerating voltage (fixed electron energy) find a pattern of sharp reflected beams from the crystal



G.P. Thomson performed similar interference experiments with thin-film samples

ELECTRON DIFFRACTION (cont)

Interpretation: similar to Bragg scattering of X-rays from crystals



Path difference:

$$a(\cos\theta_r - \cos\theta_i)$$

Constructive interference when

$$a(\cos\theta_r - \cos\theta_i) = n\lambda$$

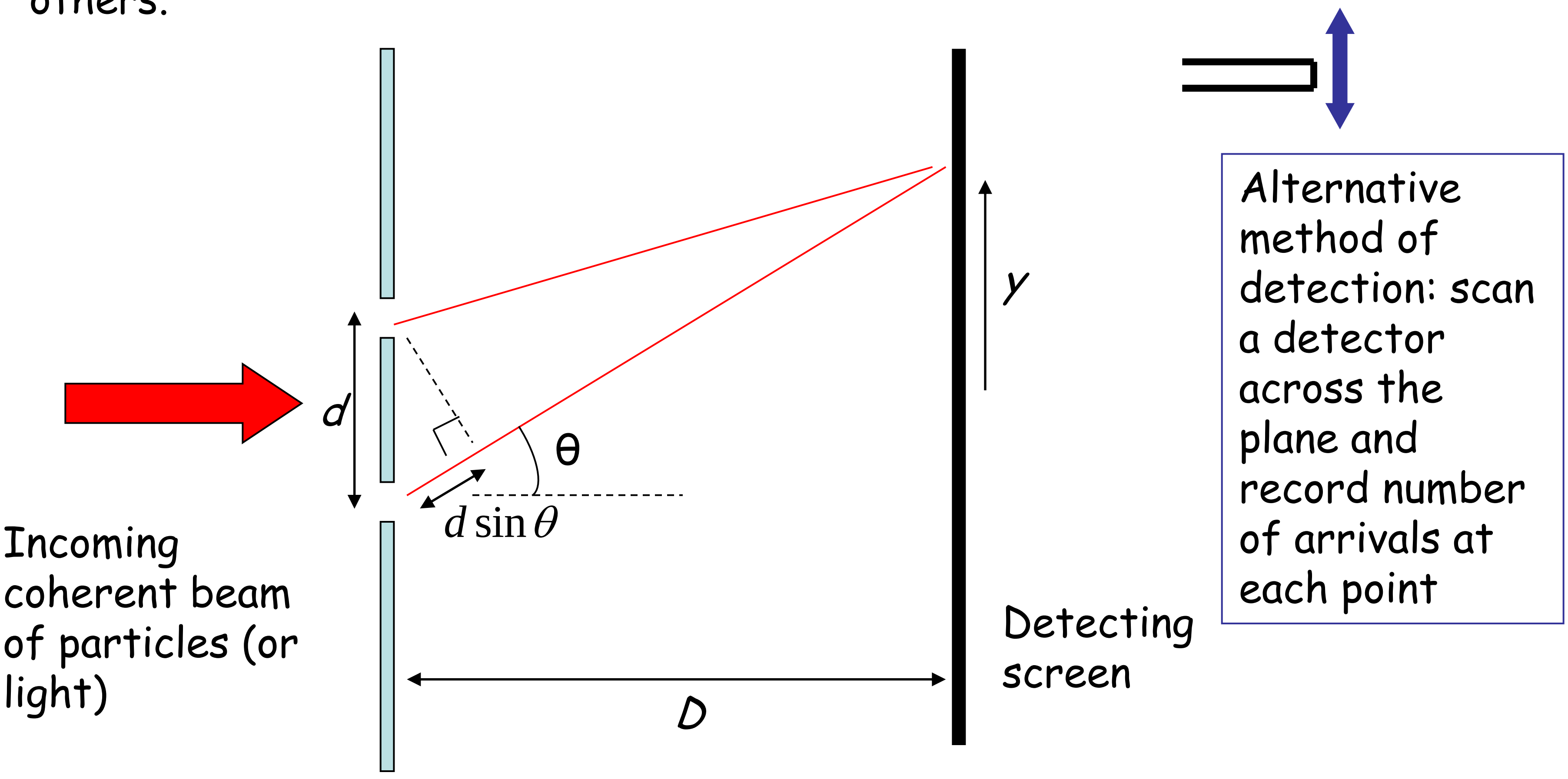
Electron scattering dominated by *surface* layers

Note θ_i and θ_r not necessarily equal

Note difference from usual "Bragg's Law" geometry: the identical scattering planes are oriented *perpendicular* to the surface

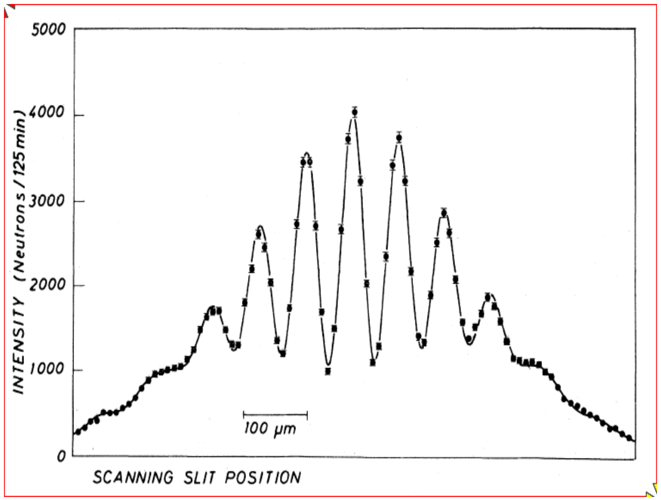
THE DOUBLE-SLIT EXPERIMENT

Originally performed by Young (1801) to demonstrate the wave-nature of light. Has now been done with electrons, neutrons, He atoms among others.

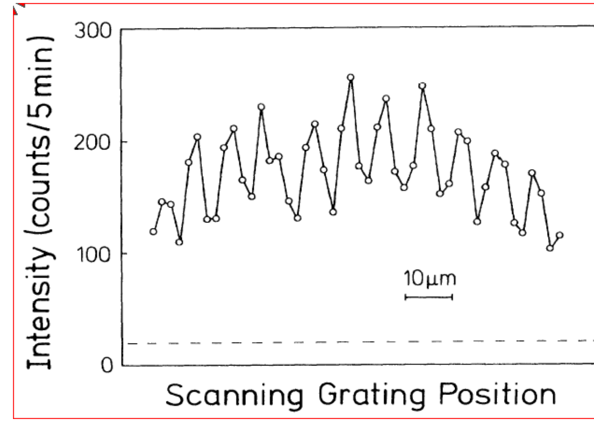


For particles we expect two peaks, for waves an interference patter

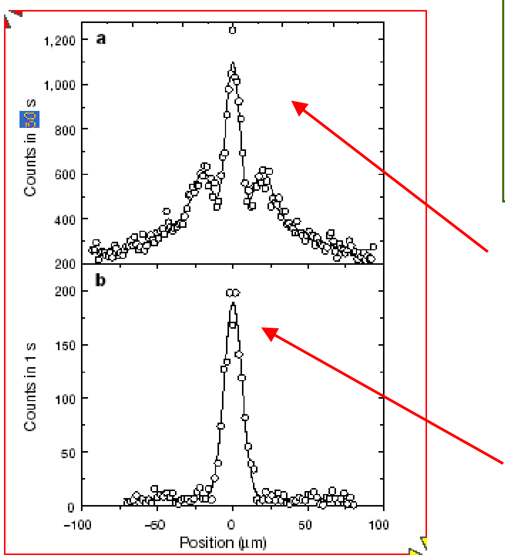
EXPERIMENTAL RESULTS



Neutrons, A Zeilinger *et al.* 1988 *Reviews of Modern Physics* **60** 1067-1073



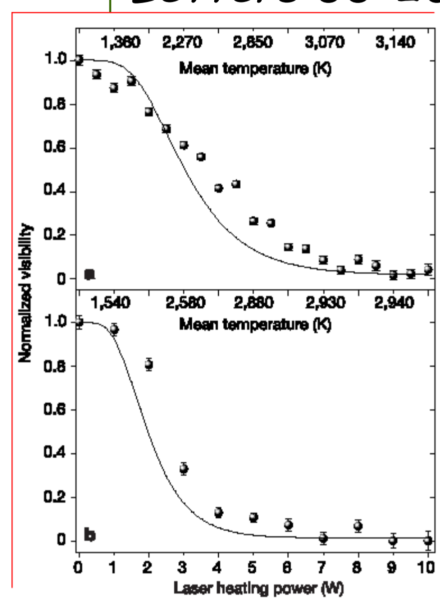
He atoms: O Carnal and J Mlynek 1991 *Physical Review Letters* **66** 2689-2692



C_{60} molecules: M Arndt *et al.* 1999 *Nature* **401** 680-682

With multiple-slit grating

Without grating



Fringe visibility decreases as molecules are heated. L. Hackermülle r *et al.* 2004 *Nature* **427** 711-714

Interference patterns can not be explained classically - clear demonstration of

DOUBLE-SLIT EXPERIMENT WITH HELIUM ATOMS
(Carnal & Mlynek, 1991,Phys.Rev.Lett.,66,p2689)

Path difference: $d\sin\theta$

Constructive interference: $d\sin\theta = n\lambda$

Separation between maxima: $\Delta y = \dfrac{\lambda D}{d}$
(proof following)

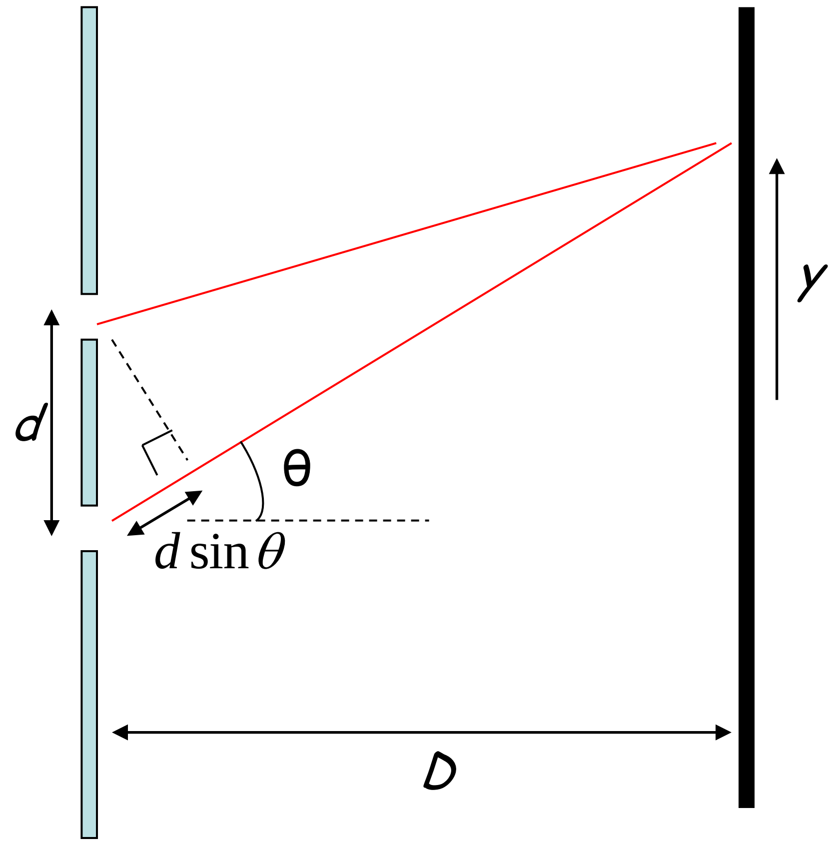
Experiment: He atoms at 83K, with *d*=8μm and *D*=64cm

Measured separation: $\Delta y = 8.2\,\mu m$

Predicted de Broglie wavelength:

$$K = \frac{3kT}{2}, \quad \text{Mass} = 4\text{m}_\text{u}$$

$$\lambda = \frac{h}{\sqrt{3MkT}} = 1.03\times10^{-10}\,\text{m}$$



Predicted separation: $\Delta y = 8.4 \pm 0.8\,\mu m$

Good agreement with experiment

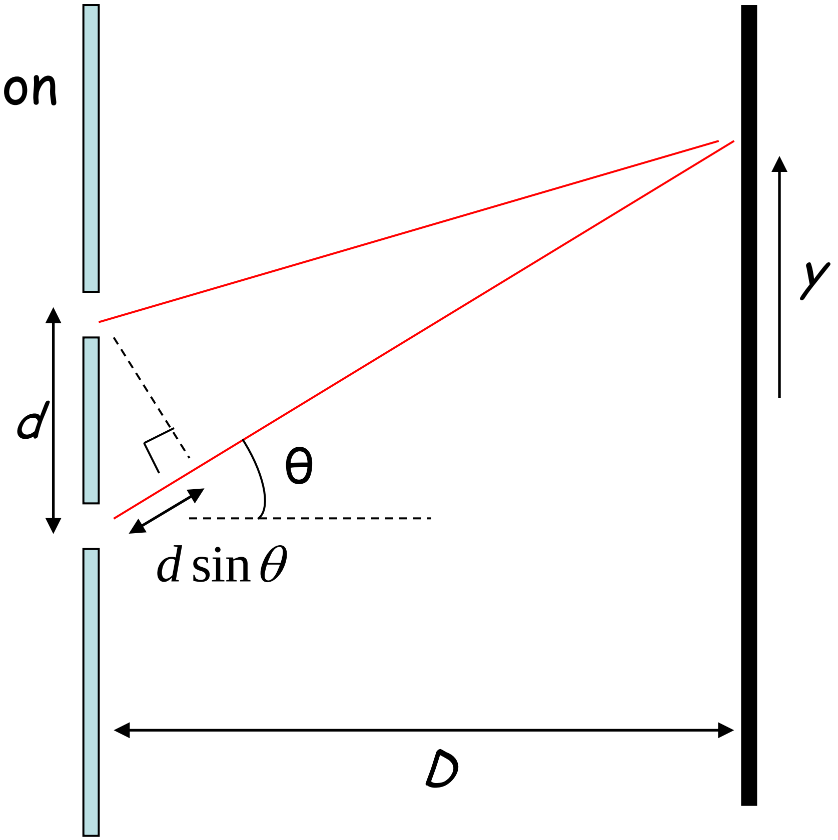
FRINGE SPACING IN DOUBLE-SLIT EXPERIMENT

Maxima when: $d\sin\theta = n\lambda$

$D \gg d$ so use small angle approximation

$$\theta \approx \frac{n\lambda}{d}$$

$$\Rightarrow \Delta\theta \approx \frac{\lambda}{d}$$



Position on screen: $y = D\tan\theta \approx D\theta$

So separation between adjacent maxima:

$$\Delta y \approx D\Delta\theta$$

$$\Rightarrow \Delta y = \frac{\lambda D}{d}$$

DOUBLE-SLIT EXPERIMENT INTERPRETATION

- The flux of particles arriving at the slits can be reduced so that only one particle arrives at a time. Interference fringes are still observed!

  Wave-behaviour can be shown by a single atom.

  Each particle goes through both slits at once.

  A matter wave can interfere with itself.

  Hence matter-waves are distinct from H_2O molecules collectively giving rise to water waves.
- Wavelength of matter wave unconnected to any internal size of particle. Instead it is determined by the momentum.
- If we try to find out which slit the particle goes through the interference pattern vanishes!

  We cannot see the wave/particle nature at the same time.

  If we know which path the particle takes, we lose the fringes .

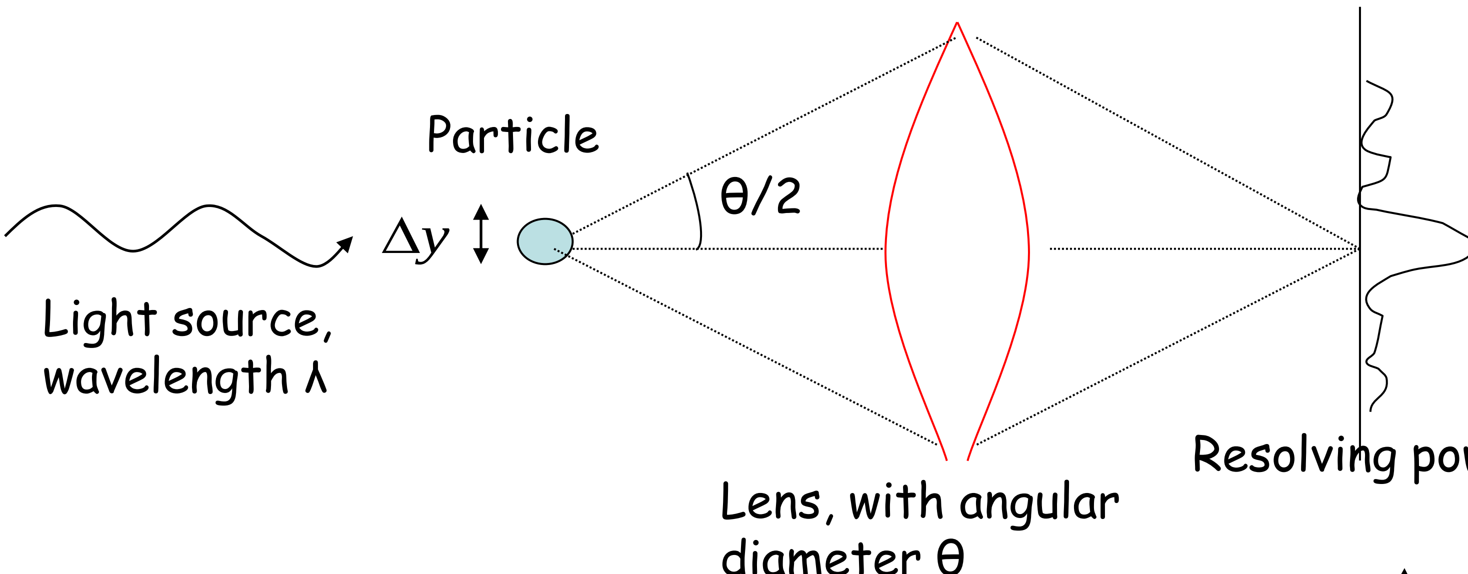
The importance of the two-slit experiment has been memorably summarized by Richard Feynman: "…a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery."

HEISENBERG MICROSCOPE AND THE UNCERTAINTY PRINCIPLE

(also called the Bohr microscope, but the thought experiment is mainly due to Heisenberg).

The microscope is an imaginary device to measure the position (y) and momentum (p) of a particle.

Heisenberg



Resolving power of lens:

$$\Delta y \geq \frac{\lambda}{\theta}$$

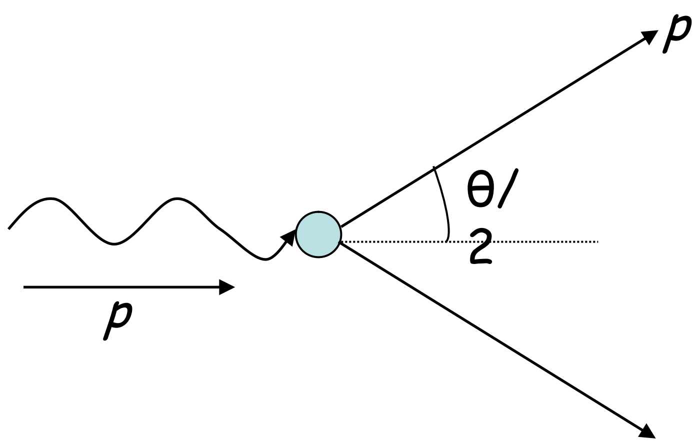
HEISENBERG MICROSCOPE (cont)

Photons transfer momentum to the particle when they scatter.

Magnitude of p is the same before and after the collision. Why?

Uncertainty in *photon* y-momentum
= Uncertainty in *particle* y-momentum

$$-p\sin(\theta/2) \le p_y \le p\sin(\theta/2)$$



Small angle approximation

$$\Delta p_y = 2p\sin(\theta/2) \approx p\theta$$

de Broglie relation gives $p = h/\lambda$ and so $\Delta p_y \approx \dfrac{h\theta}{\lambda}$

From before $\Delta y \ge \dfrac{\lambda}{\theta}$ hence

$$\Delta p_y \Delta y \approx h$$

HEISENBERG UNCERTAINTY PRINCIPLE.

Point for discussion

The thought experiment seems to imply that, while prior to experiment we have well defined values, it is the act of measurement which introduces the uncertainty by disturbing the particle's position and momentum.

Nowadays it is more widely accepted that quantum uncertainty (lack of determinism) is intrinsic to the theory.

HEISENBERG UNCERTAINTY PRINCIPLE

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\Delta y \Delta p_y \geq \hbar / 2$$

$$\Delta z \Delta p_z \geq \hbar / 2$$

HEISENBERG UNCERTAINTY PRINCIPLE.

We cannot have simultaneous knowledge of 'conjugate' variables such as position and momenta.

Note, however, $\Delta x \Delta p_y \geq 0$ etc

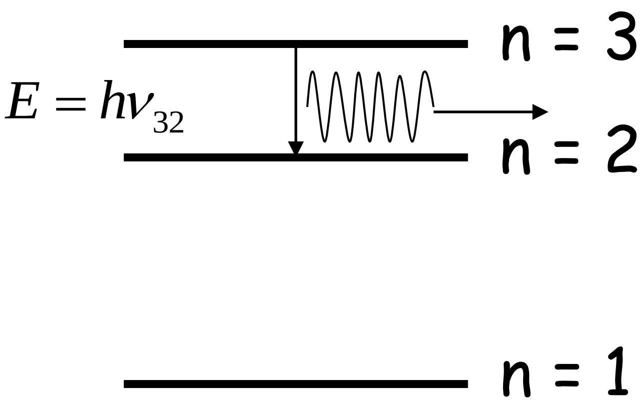
Arbitrary precision is possible in principle for position in one direction and momentum in another

HEISENBERG UNCERTAINTY PRINCIPLE

There is also an energy-time uncertainty relation

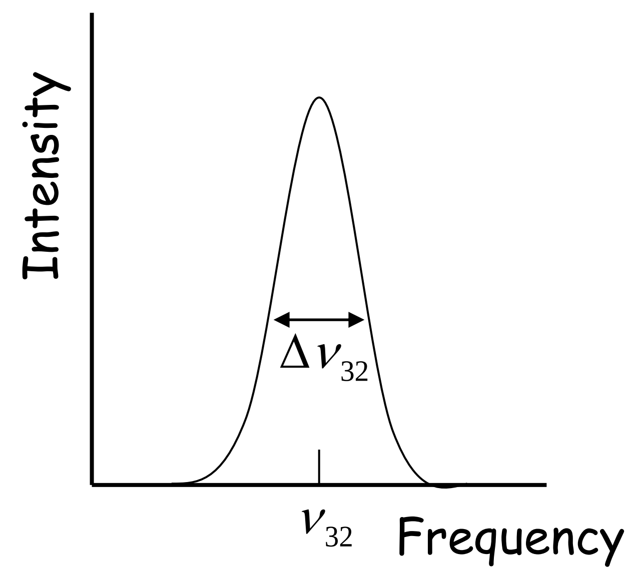
$$\Delta E \Delta t \geq \hbar / 2$$

Transitions between energy levels of atoms are not perfectly sharp in frequency.



An electron in $n = 3$ will spontaneously decay to a lower level after a lifetime of order $t \sim 10^{-8}$ s

There is a corresponding 'spread' in the emitted frequency



CONCLUSIONS

Light and matter exhibit wave-particle duality

Relation between wave and particle properties given by the de Broglie relations

$$E = h\nu \qquad p = \frac{h}{\lambda}$$

Evidence for particle properties of light
Photoelectric effect, Compton scattering

Evidence for wave properties of matter
Electron diffraction, interference of matter waves (electrons, neutrons, He atoms, C60 molecules)

Heisenberg uncertainty principle limits simultaneous knowledge of conjugate variables

$$\Delta x \Delta p_x \geq \hbar / 2$$
$$\Delta y \Delta p_y \geq \hbar / 2$$
$$\Delta z \Delta p_z \geq \hbar / 2$$